Week 10 Worksheet - Graphing

Instructions. Follow the instructions given by your TA. You are not expected to finish all the problems. :)

$f'(x)=0$ or not defined

1. What are the differences between the following terms?

   (a) Critical points VS local minimum or local maximum.

   (b) Local minimum VS global/absolute minimum.

For closed interval $[a,b]$. global ~~max~~ min can also happen at endpoints.

Max/min on closed interval

only need to check critical pts & end pts.

2. (a) Find global max / global min of $f(x) = x^{\frac{4}{5}}$ on $[-1, 2]$.

   (b) Find global max / global min of $f(x) = x^3 - 3x$ on $[0, 2]$.

(a) $f'(x) = \frac{4}{5}x^{-\frac{1}{5}} = \frac{4}{5\sqrt[5]{x}}$

$x=0$ is a critical point.

$f(-1) = \sqrt[5]{(-1)^4} = \sqrt[5]{1} = 1$

$f(0) = 0$

$f(2) = \sqrt[5]{2^4} = \sqrt[5]{16} > 1$

global max : $\sqrt[5]{16}$ $\quad (2, \sqrt[5]{16})$

global min : 0 $\quad (0,0)$

(b) $f'(x) = 3x^2 - 3 = 3(x-1)(x+1)$

$x = 1, -1$ are critical pts, $\Rightarrow$ 1 is in $[0,2]$, but -1 isn't.

$f(0) = 0$

$f(1) = -2$

$f(2) = 9$

global max = 9 $\quad (2,9)$

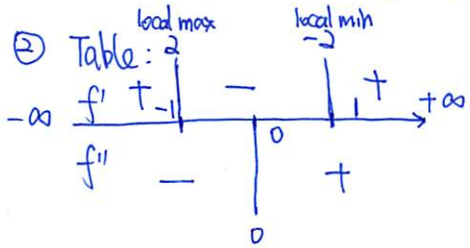
global min : -2 $\quad (1,-2)$

3. Draw the graph of $f(x) = x^3 - 3x$

① $f'(x) = 3x^2 - 3$ $\quad$ Critical pts: $x = -1, 1$

$f''(x) = 6x$ $\quad$ possible inflection pts: $x = 0$

② Table:



Or:

	$-\infty$ to -1	-1 to 0	0 to 1	1 to $+\infty$
f'	+	−	−	+
f''	−	−	+	+
shape				

③ y-values of importance pts.

$f(-1) = 2$

$f(0) = 0$

$f(1) = -2$

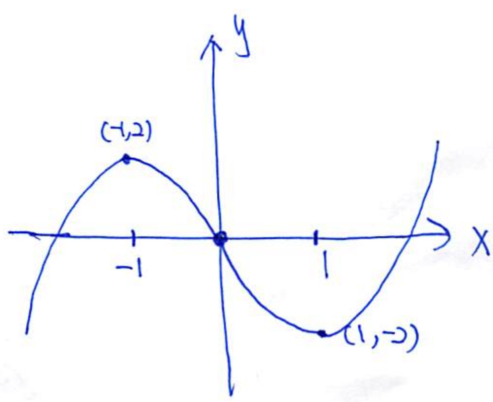
④ Asymptotes:

VA : none

HA : none

SA : none.

⑤ Sketch



4. Draw the graph of $f(x) = x + \frac{1}{x}$

① $f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2} = \frac{(x-1)(x+1)}{x^2} \Rightarrow$ critical pts : $x = -1, 0, 1$

$f''(x) = 2x^{-3} = \frac{2}{x^3} \Rightarrow$ possible inflection pt : $x = 0$

② Table.

f': $+$ | -1 (local max, -2) | $-$ | 0 | $-$ | 1 (local min, 2) | $+$

f'': $-$ | $+$

Or:

	$-\infty$ to -1	-1 to 0	0 to 1	1 to $+\infty$
f'	$+$	$-$	$-$	$+$
f''	$-$	$-$	$+$	$+$
shape	(	\	_	/

③ y-values

$f(-1) = -2$

$f(0)$: not defined

$f(1) = 2$

④ Asymptotes

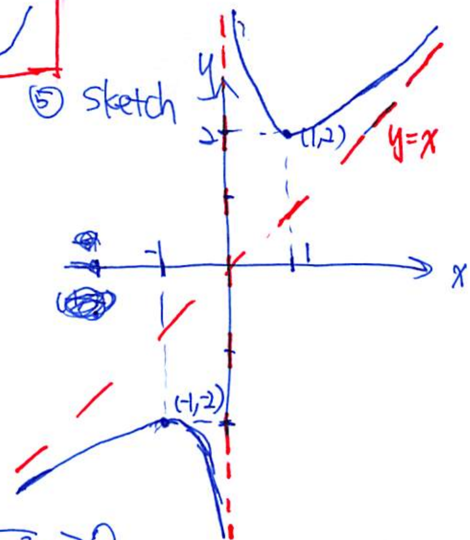
VA: $x = 0$

HA: none

SA: $m = \lim_{x\to\infty} \frac{f(x)}{x} = 1$ $\quad \boxed{y = x}$

$b = \lim_{x\to\infty} f(x) - mx = \lim_{x\to\infty} \frac{1}{x} = 0$

⑤ Sketch



5. Draw the graph of $f(x) = \sqrt{1 + x^2}$

① $f'(x) = \frac{1}{2}(1+x^2)^{-\frac{1}{2}} 2x = \frac{x}{\sqrt{1+x^2}}$ $\quad$ critical pts : $x = 0$

$f''(x) = \frac{\sqrt{1+x^2} - \frac{1}{2}(1+x^2)^{-\frac{1}{2}} 2x \cdot x}{(\sqrt{1+x^2})^2} = \frac{\sqrt{1+x^2} - \frac{x^2}{\sqrt{1+x^2}}}{(\sqrt{1+x^2})^2} = \frac{1+x^2-x^2}{(1+x^2)^{\frac{3}{2}}} = \frac{1}{(1+x^2)^{\frac{3}{2}}} > 0$

② Table

f': $-$ | 0 (local min) | $+$ $\to x$

f'': $+$

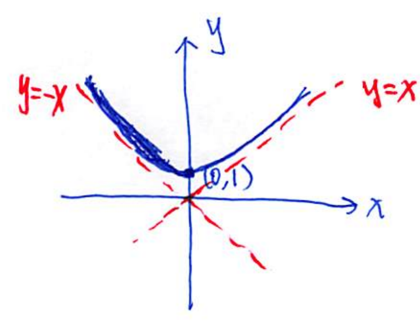
Or:

	$-\infty$ to 0	0 to $+\infty$
f'	$-$	$+$
f''	$+$	$+$
shape	_	/

③ y-values

$f(0) = 1$

⑥ Sketch :



④ Asymptotes :

VA : none

HA : none

SA : $m = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\sqrt{1+x^2}}{x} = \lim_{x\to\infty} \frac{\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}}{x} = \lim_{x\to\infty} \frac{x\sqrt{1+\frac{1}{x^2}}}{x} = 1$

$\sqrt{x^2} = |x| = \begin{cases} x & x > 0 \\ -x & x < 0 \end{cases}$

$b = \lim_{x\to\infty} f(x) - mx = \lim_{x\to\infty} \sqrt{1+x^2} - x = \lim_{x\to\infty} \frac{1}{\sqrt{1+x^2}+x} = 0$ $\quad \Rightarrow \boxed{y = x}$

$m = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{\sqrt{1+x^2}}{x} = \lim_{x\to-\infty} \frac{\sqrt{x^2}\sqrt{\frac{1}{x^2}+1}}{x} = \lim_{x\to-\infty} \frac{-x\sqrt{\frac{1}{x^2}+1}}{x} = -1$

$b = \lim_{x\to-\infty} f(x) - mx = \lim_{x\to-\infty} \sqrt{1+x^2} + x = \lim_{x\to-\infty} \frac{1}{\sqrt{1+x^2}-x} = 0$ $\quad \Rightarrow \boxed{y = -x}$